Exam 1
Chapter 1

Name: Solutions

Do not write your name on any other page. Answer the following questions. *Answers without proper evidence of knowledge will not be given credit.* Make sure to make reasonable simplifications.

Show your work!

1. (7 points) Verify that $y(x) = Ce^{-x} + x - 1$ is a solution to the differential equation

$$y' = x - y, \quad y(0) = 10,$$

and find a value of C such that $y(x)$ satisfies the given initial condition.

$y = Ce^{-x} + x - 1$

$y' = -Ce^{-x} + 1$

So $y' = x - y$

$-Ce^{-x} + 1 = x - (Ce^{-x} + x - 1)$

$= -Ce^{-x} + 1$ ✓

$y(0) = 10 = C - 1$

So $C = 11$

and $y = 11e^{-x} + x - 1$.

2. (7 points) A diesel car gradually speeds up so that for the first 10s its acceleration is given by

$$\frac{dv}{dt} = (0.12)t^2 + (0.6)t \quad (\text{ft/s}^2).$$

If the car starts from rest ($x_0 = 0$, $v_0 = 0$), find the distance it has traveled at the end of the first 10 seconds and its velocity at that time.

$$V = \int (0.12)t^2 + (0.6t)\,dt$$

$$= (0.04)t^3 + (0.3)t^2 + C$$

$$V_0 = 0 \Rightarrow C = 0$$

$$X = \int (0.04)t^3 + (0.3)t^2\,dt$$

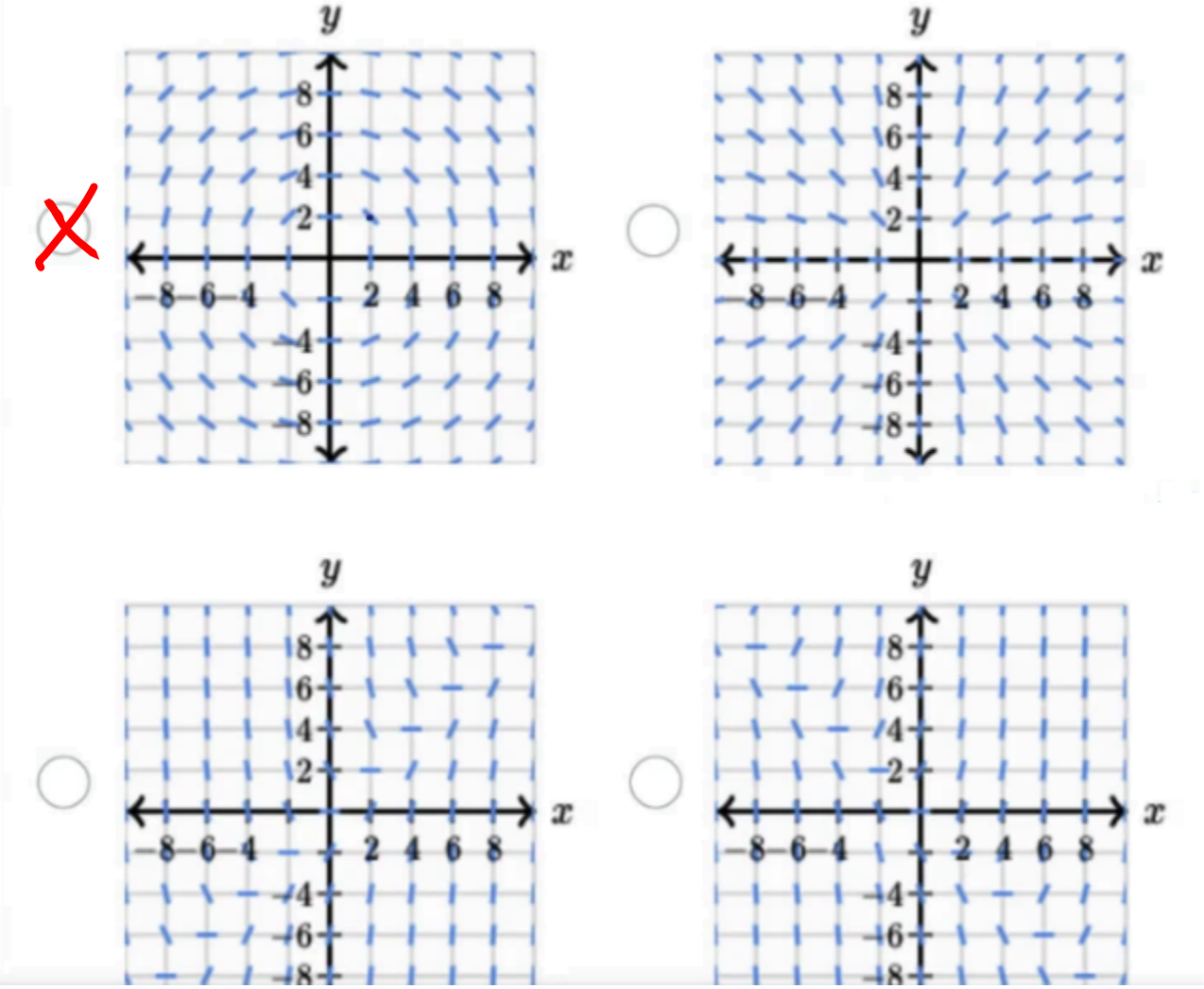
$$= (0.01)t^4 + (0.1)t^3 + C$$

$$x_0 = 0 \Rightarrow C = 0$$

$$x(10) = 100 + 100 = 200 \text{ ft}$$

$$v(10) = 40 + 30 = 70 \text{ ft/s}.$$

3. (6 points) Which slope field is generated by the differential equation $\frac{dy}{dx} = -\frac{x}{y}$?



$y' = -\frac{x}{y}$, so $y'(1,1) = -1$.

4. (10 points) Find the general solution to the differential equation

$$\frac{dy}{dx} + y^2 \sin x = 0.$$

$$\frac{dy}{dx} + y^2 \sin x = 0$$

$$\frac{dy}{dx} = -y^2 \sin x$$

$$\int \frac{dy}{y^2} = \int -\sin x \, dx$$

$$-\frac{1}{y} = +\cos x + C$$

$$y = \frac{1}{-\cos x + C}$$

5. (10 points) Find the particular solution to the differential equation on the given domain

$$y' + \frac{2}{t}y = \frac{\cos t}{t^2}, \quad y(\pi) = 0, \ t > 0.$$

$P(t) = e^{\int \frac{2}{t} dt} = e^{2\ln t} = t^2.$

So $t^2 y = \int t^2 \cdot \frac{\cos t}{t^2} dt$

$= \int \cos t \, dt$

$= \sin t + C$

Thus

$y = t^{-2}(\sin t + C)$

$y(\pi) = 0$, so $C = \pi^2$

and $y = t^{-2}(\sin t + \pi^2)$.

6. (10 points) Use the method of exact equations to solve the differential equation

$$(e^x \sin y + \tan y)dx + (e^x \cos y + x \sec^2 y)dy = 0.$$

$\int e^x \sin y + \tan y \, dx = e^x \sin y + x \tan y + C(y)$

$\int e^x \cos y + x \sec^2 y \, dy = e^x \sin y + x \tan y + C(x)$.

Therefore

$F(x,y) = e^x \sin y + x \tan y = C$.